Indian J. Phys. 73A (5), 681–690 (1999)

IJP A
— an international journal

Practical approach for determination of the optical potentials at different energies

N M Eldebawi
Department of Physics, Faculty of Science, Zagazig University, Zagazig, Egypt

Received 24 September 1998, accepted 5 May 1999

Abstract : We present as a direct consequence for the need of obtaining an optical potential free from parameter ambiguities that describes light-heavy ion scattering, a simple and practical procedure to determine the optical potential which fits elastic scattering data The method is a two step procedure First, parameterization of the phase shifts to analysis the angular distribution data; secondly, Glauber's eikonal approximation is used to derive the optical potentials corresponding to the scattering of strongly absorbed nuclear projectiles The procedure is successfully applied to the analysis of experimental data of ^{6}Li on ^{28}Si reaction in a wide energy range from 46 to 318 MeV.

Keywords : Optical potential, elastic scattering, Glauber's eikonal approximation

PACS Nos. : 24.10 Ht, 25.60 Bx, 25.70.Bc

1. Introduction

Conventional optical model analysis does not allow a unique determination of the parameters of the optical potentials of strongly absorbed particles. The assumption that optical potential for a composite projectiles is just the sum of the optical potentials of its constitute nucleons [1], unfortunately, this assumption is not satisfactory for the calculation of the optical potentials of heavier projectiles [2]. Johnson and Soper [3] suggest an alternative method for the calculation of the optical potential of composite projectiles. Such projectiles are strongly absorbed by the target nucleus and their scattering is successfully described by the nuclear diffraction model [4]. One alternative methods for calculating the optical potentials is the two-step method, (matrix $S(l)$ search followed by $S(l)$ to $V(r)$), whereby the scattering matrix is determined by a Phase-shift search and then subjected to fixed energy $S(l)$ to $V(r)$ inversion [5]. Cooper *et al* [6] used an S-matrix fitting procedure then followed by $S(l)$ to $V(r)$ inversion. However, two-step phenomenology used to establish the potential by inversion $S(l)$ to $V(r)$ presents serious ambiguities even for exact fits with very precise data; difficulties also arise in determining $S(l)$ from the observable $\sigma(\theta)$. Previous works [7,8] provide examples of how discontinuous ambiguities arise by showing that the very different type potentials each giving the same rainbow-like angular distribution, but in fundamentally different ways.

Aims of the present work are to (i) Draw attention towards a simple and practical method to determine the optical potentials in terms of diffraction model, (ii) Explore the ability of the strong absorption model in describing the elastic scattering of projectiles of mass intermediate between light and heavy ions and (iii) Using the elastic scattering of ^{6}Li on ^{28}Si data, the nuclear potential is determined and the energy dependence of ^{6}Li on ^{28}Si optical potentials is discussed.

2. The method

Elastic scattering of heavy ions has been successfully described in the framework of the strong absorption model [4,9]. The basic assumption of this model is to replace the scattering matrix elements η_l by a function of orbital angular momentum l and regarding the latter as a continuous variable *i.e.*, according to this model the l-wave scattering matrix is written as the product of the Coulomb scattering $\eta_l = \exp\,(2i\delta_l)$, where δ_l is the Coulomb phase shift, and η_l the nuclear scattering coefficients. Then considering η_l as a continuous function of l which takes vanishingly small values for small l and monotonically increases with l until it approaches unity, the contribution from small values of l will be negligible except for very small angles. Accordingly, one uses the asymptotic expressions for the Legendre polynomials. When the Coulomb interaction $n = Z_1Z_2e^2/\hbar v$ is small, the Coulomb phase-shifts smoothly vary with l. They can be approximated by the first two terms in a power series of $l - l_0$, where l_0 is the cut off orbital angular momentum, replacing the summation over l in the expression for the scattering by integration [10] or performing the Watson-Sommerfeld transformation and counting only the contribution from the two poles nearest to the real axis of the function [11].

Ericson [12] suggested a convenient parameterization of the phase shifts in studying the scattering function in the complex angular momentum plane, as the following :

$$\exp(2i\delta_l) = \left[1 + \exp\left(\frac{l_0 - l - i\lambda}{\Delta}\right)\right]^{-1} \tag{1}$$

Using the Watson-Sommerfeld transformation, one obtains the following analytical expression for scattering cross section :

$$\sigma(\theta) = N\,\mathrm{cosec}(\theta)\exp(-2\pi\Delta\theta)\left[\cos^2\left(l_0 + 1/2\right)\theta + \pi/4 + \mathrm{arct}\frac{\pi\Delta}{l_0 + 1/2} + \sinh^2(\pi\Delta\theta_c)\right], \tag{2}$$

where $$N = 8\pi\left(\frac{\Delta}{k}\right)^2 \sqrt{(l_0 + 1/2)^2 + (\pi\Delta)^2}\, \exp(2\lambda\theta_c),$$

$$\theta_c = 2\arctan\frac{n}{l_0 + 1/2},$$

$$n = Z_1 Z_2 e^2/\hbar v,$$

where θ_c is the Coulomb deflection angle, Z_1 and Z_2 being the projectile and target atomic numbers, and v is their relative velocity. One finds that eq. (2) reflects the exponential decrease of the cross section as the scattering increases and reproduces the oscillations characterizing the diffraction scattering pattern. Nevertheless, we shall not expect that eq. (2) yields a complete agreement to the experimental data where important effects, namely the reflection and refraction of the partial waves at the nuclear surface are not taken into account by neglecting the nuclear phase shifts. Simbel and Abul Magd [13] modified eq. (2) by taking the real nuclear phase shifts into account as :

$$\sigma(\theta) = N\operatorname{cosec}(\theta)\exp(-2\pi\Delta\theta)\Big[\cos^2\Big((l_0 + 1/2)\theta + \pi/4 + \operatorname{arct}\frac{\pi\Delta}{l_0 + 1/2}\Big) + \sinh^2(\pi\Delta\theta_c - \lambda\theta)\Big]. \tag{3}$$

Eq. (3) differs from eq. (2) by the fact that the $\sinh^2$ term now depends on θ, and term vanishes at $\theta = \theta_c\, \pi\Delta/\lambda$.

The determination of the parameters from the experimental data is quite a simple task. Plotting the experimental cross section multiplied by $\sin\theta$ on a semi-logarithmic scale, one finds an approximate value for the parameter Δ from the slope of the curve and for the parameter l_0 from the period of the diffraction oscillations. Using these values, one evaluates the quantity θ_c and then calculates the parameter λ from the position of the two deepest minima. Then, one plots the quantity $\sigma_m \sin\theta/\sinh^2(\pi\Delta\theta_c - \lambda\theta)$ on a semi-logarithmic scale, where σ_m is the experimental cross section at the diffraction maxima, and determines a new value for the parameter Δ. With the new value of Δ, one finds a new value of λ and so on. Repeating this procedure until a self-consistent set of parameters is achieved, the values of the parameters l_0, λ, Δ are obtained. As we have seen, each of the above mentioned parameter is responsible for a certain behavior of the angular distribution. Therefore the extracted parameters are unique, in contrast to the parameters of the optical model potential which are highly ambiguous for the scattering of strongly absorbed particles.

Comparison between the values of $\sigma(\theta)$ obtained using this expression, with those obtained by the exact summation of the partial waves with δ_l given by eq. (1), suggests that this expression is accurate for angles $\theta > \theta_c$. One can notice that :

(a) The cosine function in eq. (3) is responsible for the oscillation pattern of differential cross section. The average period of oscillation is equal to $\Theta = \pi/(l_0 + 1/2)$.

(b) The heights of the diffraction minima are proportional to the quantity $\sinh^2(\pi\Delta\theta_c - \lambda\theta)$. The deepest pair of minima thus occurs near the angle $\theta_{\min} = \pi\Delta\theta_c/\lambda$.

(c) At large value of θ, the contribution of the oscillating cosine function decreases compared to that of the hyperbolic function. Therefore eq. (3) can be approximated by an exponential function and takes the form :

$$\sigma(\theta)\sin\theta \approx \frac{N}{4}\exp(-2\pi\Delta\theta_c)\exp\left[2(\lambda - \pi\Delta)\theta\right], \quad \text{when } \theta > \pi\Delta\theta_c/\lambda$$

Thus, the slope of the exponential decay of the differential cross section data fall off at large angles and is approximately given by $2(\lambda - \pi\Delta)$.

We now consider the inversion procedure $S(l) \rightarrow V(r)$ using Glauber's eikonal approximation [14]

$$f(\theta) = -ik\int_0^{\infty} J_0(2kb\sin\theta)\left\{\exp 2i\chi(b) - 1\right\} b\,db, \tag{4}$$

where b is the impact parameter, $j_0(x)$ is the Bessel function, and $\chi(b)$ is the thickness profile defined by

$$\chi(b) = \frac{1}{\hbar v}\int_0^{\infty} \frac{V(r)}{\sqrt{r^2 - b^2}}\, r\,dr, \tag{5}$$

where $V(r)$ is the optical potential and v is the incident velocity. Using the semi-classical relation $l \approx kb$ and regarding l as continuous variable. The function $\chi(b)$ is identified with the phase shifts δ_l given by eq. (1); eq. (5) becomes an Abel integral equation [15] and has the solution :

$$V(r) = \frac{\hbar v}{2\pi}\frac{1}{r}\frac{d}{dr}\int_r^{\infty} \frac{\chi(b) b\,db}{\sqrt{b^2 - r^2}}, \tag{6}$$

$$V(r) = \frac{\hbar v}{\pi a i}\int_0^{\infty} \frac{du}{\sqrt{r^2 + u^2}\left[1 + \exp\left(\dfrac{\sqrt{r^2 + u^2} - R_0 + i\rho}{a}\right)\right]}. \tag{7}$$

In order to obtain the optical potentials, eq. (7) has been separated into two parts, the real part :

$$\mathrm{Re}(V) = \frac{\hbar v}{\pi a} \times \int_0^{\infty} \frac{\exp\left(\dfrac{\sqrt{r^2+u^2} - R}{a}\right)\sin(\rho/a)}{\sqrt{r^2+u^2}\left(1 + \exp 2\left(\dfrac{\sqrt{r^2+u^2} - R}{a}\right) + 2\exp\left(\dfrac{\sqrt{r^2+u^2} - R}{a}\right)\cos(\rho/a)\right)}, \tag{8}$$

and the imaginary part of the optical potential :

$$\mathrm{Im}(V) = \frac{\hbar v}{\pi a} \times \int_0^{\infty} \frac{1+\exp\left(\frac{\sqrt{r^2+u^2}-R}{a}\right)\cos(\rho/a)}{\sqrt{r^2+u^2}\left(1+\exp 2\left(\frac{\sqrt{r^2+u^2}-R}{a}\right)+2\exp\left(\frac{\sqrt{r^2+u^2}-R}{a}\right)\cos(\rho/a)\right)} du \cdot \quad (9)$$

The integration in eqs. (8) and (9) can be carried out numerically and the optical potential can be obtained.

3. Application to ^{6}Li on ^{28}Si

Elastic scattering of the stable isotopes of Li has been a subject of study for several decades, the study of ^{6}Li elastic scattering and the phenomenological description of differential cross section data are of value because of it being the lightest projectile in the transition region between that characteristic of light-ions and heavy-ions elastic scattering. Therefore, we find motivations to choose ^{6}Li as an application for the present method in the region of heavy-light ion elastic scattering in order to determine the optical potentials.

We have made two sets of calculations for the elastic scattering of ^{6}Li on ^{28}Si over a wide energy range in order to determine the optical potential. We have used eq. (3) and unique parameters : the cut off angular momentum l_0 which is related to the nuclear radius R; the parameter Δ which is related to the surface diffusness a; and the parameter λ which characterizes the reflection and refraction at the nuclear surface, has been obtained by means of the following semiclassical relations [16].

$$l_0 = kR, \qquad \Delta = ka,$$

$$k\rho = \frac{\lambda(l_0+1/2)}{\sqrt{n^2+(l_0+1/2)^2}}, \qquad kR = n + \sqrt{n^2+(l_0+1/2)^2} \cdot \quad (10)$$

The obtained parameters are listed in Table 1, and the differential cross sections for energies 46.0, 99.0, 135.1, 154.0, 210.0 and 318.0 MeV are calculated and the agreement

Table 1. The parameters obtained from eqs (3) and (10) which used to fit the angular distribution for elastic scattering of ^{6}Li on ^{28}Si.

E (MeV)	Δ	λ	l_0	R	ρ	a
46.0	1.70	1.88	18.20	7.076	0.567	0.566
99.0	2.35	1.95	26.50	6.511	0.442	0.533
135.1	2.70	2.50	31.00	6.399	0.485	0.525
154.0	3.50	3.55	34.00	6.523	0.646	0.637
210.0	3.20	3.60	39.00	6.334	0.561	0.499
318.0	5.50	6.50	48.00	6.155	0.823	0.697

with experimental data are shown in Figure 1. The experimental data are taken from References [17–22].

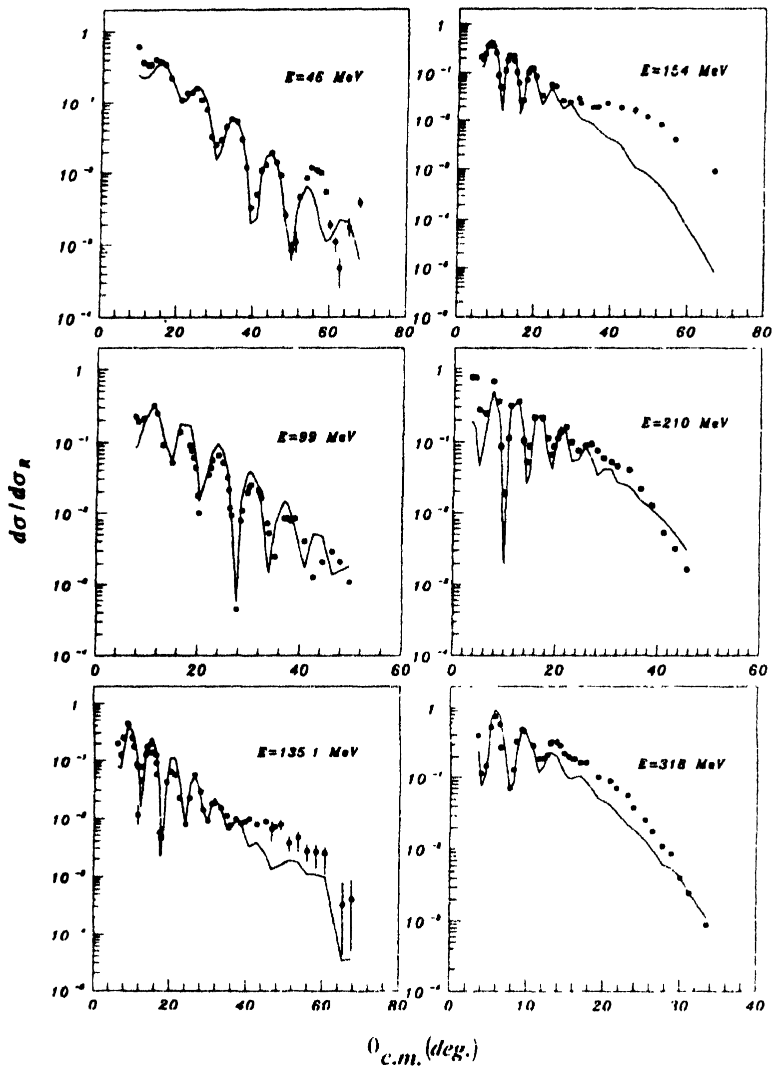


Figure 1. The differential cross section for elastic scattering of ^6Li on ^{28}Si, in the energy range from 46–318 MeV. The dots are the experimental data. The solid lines represent the result of the calculation using eq. (3).

Using calculated parameters, the integration in eqs. (8) and (9) can be carried out numerically, the real and imaginary parts of the nuclear potential are obtained, the results are illustrated in Figures (2, 3) which is compared with those of the Woods-Saxon potential.

4. Discussion and conclusion

Comparing the method used here with other methods, we find that in most of them, the problem of ambiguous parameters in the determination of the S-matrix by fitting the experimental angular distribution, is still a subject of several debates. Here "Ericson parametrization" involves three parameters, each one reflects a specific aspect of the experimental distribution. Therefore, we say that : the parameters obtained are unique and the method of obtaining them is simple and easy task.

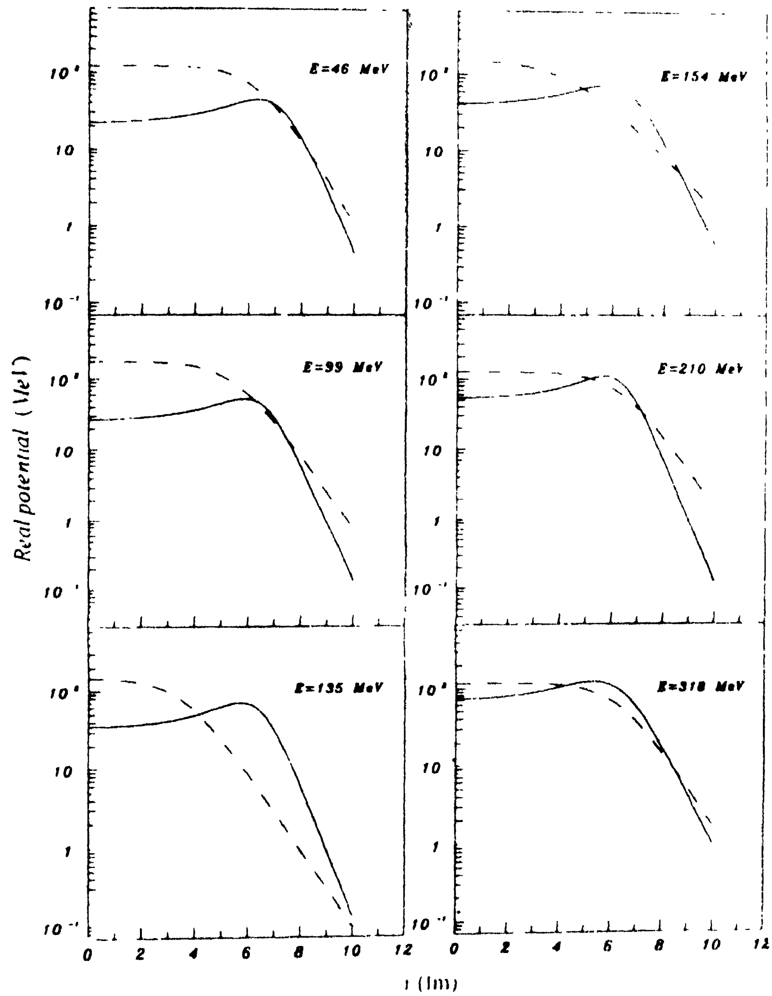


Figure 2. The real part of optical potential for elastic scattering of ^{6}Li on ^{28}Si, in the energy range from 46–318 MeV The solid lines represent the result obtained by the method used in this paper. The dashed lines represent Saxon potentials

We see that the result of our calculation for real and imaginary part of nuclear potential, using Glauber approximation is in good agreement with Wood-Saxon calculation [*see* Figure (2) for the real part, and Figure (3) for the imaginary part of the potential]. Brandan *et al* [23] discuss the failure of the Glauber approximation for light-heavy ions, but the results of this work and also the previous calculations for $^{12}C + ^{12}C$ and $^{16}O + ^{12}C$

systems [24] which cover a wide energy range (139.5–2400 MeV), and in which the Glauber approximation is also used, show agreement with other authors and show disagreement with Brandan effect.

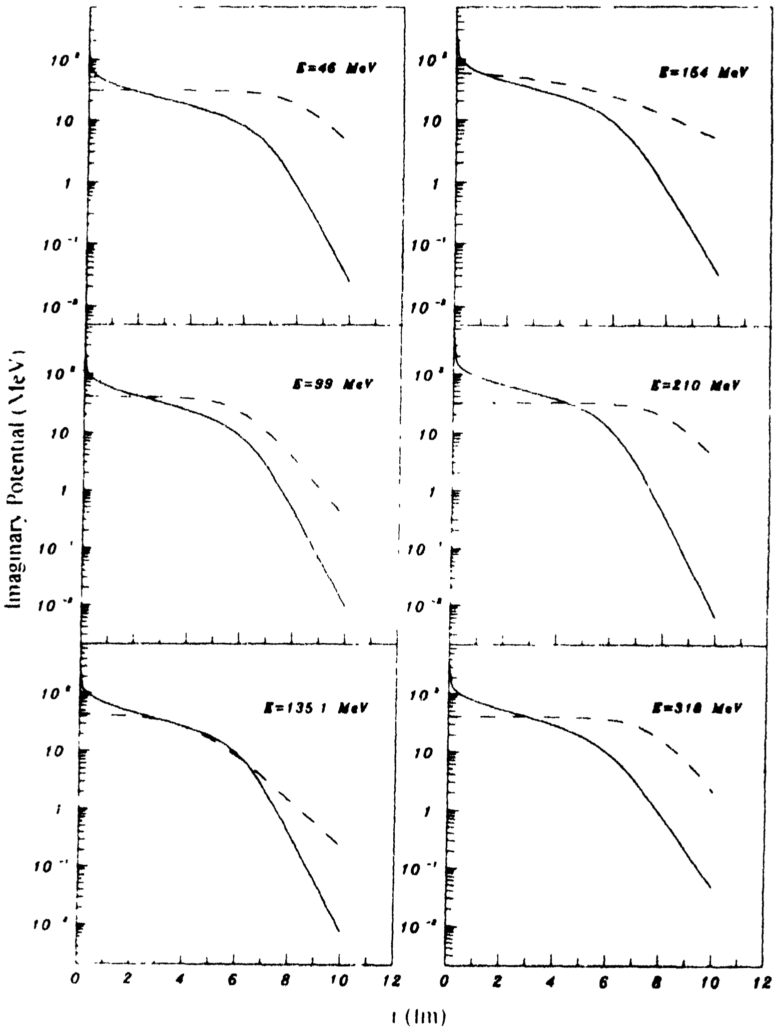
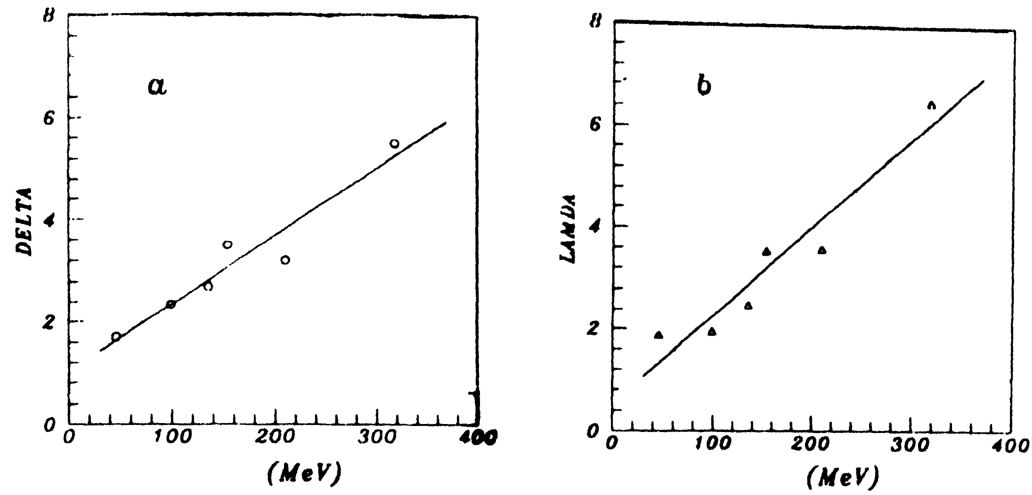


Figure 3. Comparison of the imaginary parts of the potential for the reaction in the energy range from 46–318 MeV The solid lines represent the result of the method used in this paper, the dashed lines represent Woods-Saxon Potentials

We can notice from Table 1 that λ/Δ increases with increasing the energy (expect for the low energy 46 MeV) where λ/Δ is proportional to the ratio of the real to the imaginary parts of the corresponding optical potential which in agreement with Brandan and McVoy [25]. We find that the energy dependence of Δ is defined by linear relation : Δ = 0.0133 E + 1.023 [Figure (4a)], also the parameter λ is energy dependent and is defined by λ = 0.0175 E + 0.526 as illustrated in Figure (4b), and the relation between the parameters Δ which is related to the surface diffusness a and the parameter λ, can be expressed by the relation : $\Delta = 0.748\ \lambda + 0.669$ [*see* Figure (4c)]. As seen from Table 1, l_0 increases with increasing energy.



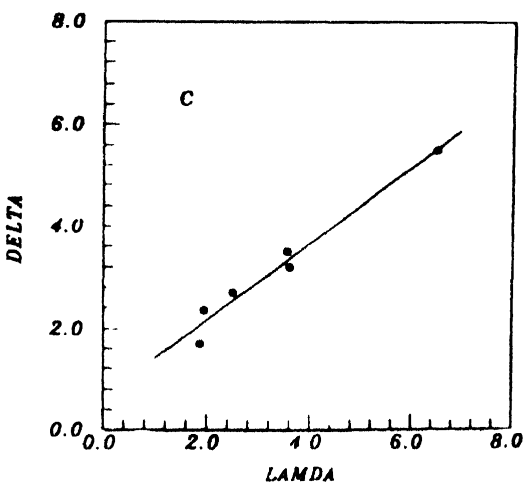


Figure 4. The energy dependence the parameters obtained (a) Delta *vs* energy, (b) Lambda *vs* energy and (c) The relation between delta and lambda, as illustrated.

In summary, the present formalism, although starts with a parameterized S-matrix as already done by the McEwan *et al* [26] and Cooper [6], it presents a method for determining unique parameters. The analytical eq. (3) is deduced, assuming strong absorption, allowing one to identify each parameter with specific feature of the angular distribution data. The inversion $S(l)$ to $V(r)$ procedure is further carried out analytically, yielding a simple expression of the optical potential in terms of these uniquely defined parameters.

References

[1] J R Rook *Nucl. Phys.* **61** 219 (1965)

[2] T Z Fliessbach *Z. Phys.* **238** 329 (1970)

[3] R C Johnson and P J R Soper *Phys. Rev.* **C1** 976 (1970)

[4] J S Blair *Phys. Rev.* **95** 1218 (1954); W E Frahn and R H Venter *Ann. Phys.* **24** 135 (1964)

[5] A M Kobos, R S Mackintosh and J R Rook *Nucl Phys* **A389** 205 (1982); S G Cooper and R S Mackintosh *Phys. Rev.* **C43** 1001 (1991)

[6] S G Cooper, M A McEwan and R S Mackintosh *Phys Rev.* **C45** 770 (1992)

[7] Z Basrak and F Auger *Nucl. Phys.* **A441** 150 (1985); K Chadan and P C Sabatier *Inverse Problems in Quantum Scattering Theory* 2nd edn (Berlin Springer) (1989); S G Cooper and R S Mackintosh *Nucl Phys.* **A576** 308 (1994); C Steward, H Fiedeldey, K Amos and L J Alley *Phys Rev.* **C51** 836 (1995)

[8] M E Brandan, K W McVoy and G R Satchler *Phys. Lett* **B281** 185 (1992)

[9] J A McIntyre, K H Wang and L C Becter *Phys Rev* **117** 1337 (1960)

[10] W E Frahn and R H Venter *Ann. Phys.* **24** 243 (1963), R H Venter *Ann. Phys.* **25** 405 (1964)

[11] E V Inopin *JETP (Soviet Phys.)* **21** 1238 (1965); J Hogaasen *Nucl. Phys.* **A90** 261 (1967)

[12] T E O Ericson in *Preludes in Theoretical Physics* (Amsterdam : North Holland) p 321 (1965)

[13] M H Simbel and A Y Abul Magd *Nucl. Phys* **A120** 152 (1968)

[14] R J Glauber in *Lectures in Theoretical Physics* eds W E Britain and L G Dunham (New York Interscience) p 1 (1959); A Y Abul Magd, M El Nadi and M H Simbel *Phys. Lett.* **B34** 566 (1971)

[15] T Y Wu and T Ohmura *Quantum Theory of Scattering* (New Jerey · Prentice-Hall) (1962)

[16] W E Frahn in *Treatise on Heavy Ion Reaction* ed A D Bromley (New York · Plenum) Vol. **1** p 135 (1988)

[17] J Cook *Nucl. Phys.* **A288** 152 (1982)

[18] P Schwandt, W W Jacobs, M D Kaitchuck, P P Singh, W D Ploughe, E D Becchetti and J Jänecke *Phys Rev.* **C24** 1522 (1981)

[19] R M DeVriess, D A Goldberg, J W Watson, M S Zisman and J G Gramer *Phys Rev Lett.* **C39** 450 (1977)

[20] P Schwandt, S Kailas, W W Jacobs, M D Kaitchuck, W D Ploughe and P P Singh *Phys. Rev* **C21** 1656 (1980)

[21] A Nadasen, M McMaster, M Fingal, J Tavormina, P Schwandt, J S Winfield, M F Mohar, F D Becchetti, J W Jänecke and R E Warner *Phys. Rev.* **C39** 536 (1985)

[22] A Nadasen, T Stevens, J Farhat, J Brusoe, P Schwandt, J S Winfield, G Yoo, N Anantaraman, F D Becchetti, J Brown, B Hotz, J W Jänecke, D Roberts and R E Warner *Phys Rev* **C47** 674 (1993)

[23] M E Brandan, H Chehime and K W McVoy *Phys. Rev.* **C55** 1353 (1997)

[24] N M Eldebawi and M H Simbel *Phys. Rev.* **C53** 1 (1996)

[25] M E Brandan and K W McVoy *Phys. Rev.* **C55** 1362 (1997)

[26] M A McEwan, S G Cooper and R S Mackintosh *Nucl. Phys.* **A552** 401 (1993)